University of Calcutta
Mode of Examination: Online
M.Sc. Semester – I Examination, 2021
Subject: Computer Science
Paper Code & Name: CSMC101 (Mathematics for Computing)

Full Marks: 70

Date: 03.03.2022 **Time and Duration: 12.00 PM – 3:00 PM (3:00 Hours)**

<u>Please note the following instructions carefully:</u>

- **Promise not to commit any academic dishonesty.**
- **Marks will be deducted if the same/similar answers are found in different answer scripts.**
- **Candidates are required to answer in their own words as far as applicable.**
- **The figures in the margin indicate full marks.**
- **Each page of the answer-scripts should have your University Roll # on the right-top corner.**
- **The name of the scanned copy of the answer script will be of the following format:**
  CSMC101-MC-University-1oll-Number.pdf
  (Example: CSMC101-MC-C91-CSC-2110xy.pdf)
- **The subject of the mail should be the file name only.**
- **The scanned answer script is to be sent to cucse2020@gmail.com**
- **The file should have the top page (Page #1) as an index page, mentioning all page numbers against the answer of each question number.**
- **The answer script may not be accepted after the scheduled time.**

Group-A

1. Answer any five from the following questions: **[2x5=10]**

a. Find the cofactors of the elements of the 2[nd] row of A and hence find the determinant value of A = $\begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

b. Given two events A and B, such that Pr(A)=0.4, Pr(B)=0.6, and Pr (AUB)=.75, find the probability that exactly one of the events will occur.

c. Find the rank of the matrix $\begin{pmatrix} 6 & 12 & 6 \\ -1 & -2 & 3 \\ 5 & 10 & 5 \end{pmatrix}$

d. Find the algebraic and geometric multiplicity of an Eigenvalue 2 for the following matrix.

$$\begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}$$

e. If the number of internal vertices in a binary tree is 15, find the number of pendant vertices.

f. If four dice are rolled, what is the probability that each of the four numbers that appear will be different?

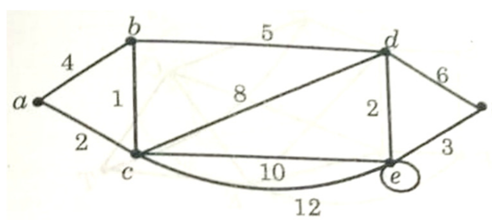
Group-B

2. Answer any five from the following questions: [4x5=20]

a. Verify that whether the set of vectors {(1, 2, 2), (1, -1, 2), (1, 0, 1)} forms a basis in R^3

b. If a linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ is defined by T(x, y) = (2x + 3y, x – y), then find the matrix of T corresponding to the standard order basis {(1,0), (0,1)}. Also, find the nullity of T and thereby determine if T^{-1} exists. If T^{-1} exists, then find T^{-1}.

c. Using Dijkstra's Algorithm, find the shortest path and the length of the shortest path from the vertex a to t:



d. Find the minimum number of edges in a simple graph with n vertices and k completely connected components where k<n.

e. Suppose three letters are put randomly into three envelopes. Let X denote the random variable such that X=k if exactly k of the letters are put in correct envelopes. Find the probability function (probability mass function) of X.

f. In a certain city, 30 percent of the people are Conservatives, 50 percent are Liberals, and 20 percent are Independents. Records show that 65 percent of the Conservatives voted in a particular election, 82 percent of the Liberals voted, and 50 percent of the Independents voted. If a person in the city is selected at random and it is learned that she did not vote in the last election, what is the probability that she is Liberal?

3. a) State and prove Euler's theorem for the planar graph. Hence prove that $e \le 3n-6$; where e and n denote the number of edges and vertices respectively of a simple connected planar graph with n>2 **[2+3+3=8]**

b) Construct the graph or Di-graph corresponding to the following incidence matrix **[2]**

$$\begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 1 \\ -1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

4. a) Consider the following recurrence relation: $a_n = 11a_{n-1} - 39a_{n-2} + 45a_{n-3}$. Write down the order of this recurrence relation and also discuss the linearity and homogeneity. Hence solved this relation with initial conditions $a_0=5$; $a_1=11$, and $a_2=25$.

[2+3=5]

b) Suppose a point is chosen at random on a stick of unit length, and the stick is broken into two pieces at that point. Find the expected value of the length of the longer piece.

[5]

5. a) Using generating function, solve the following recurrence relation.

$$a_n - 7a_{n-1} + 10a_{n-2} = 2; \forall n > 1 \, and \; a_0 = 3, a_1 = 3$$

[5]

b) Given two random variables, X and Y, their joint pdf is given by

$f(x, y) = \frac{1}{\pi}$, for $0 \le x^2 + y^2 \le 1$ and $f(x, y) = 0$, otherwise.

Find the marginal pdf of X. Also find the cdf F(x) where F(x)=Pr(X≤ x). **[5]**

6. a) If possible then solve the following by Cramer's rule:

3x + y + z = 4 ; x – y + 2z = 6; x + 2y – z = -3 **[4]**

b) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 2 & 0 & 1 \end{bmatrix}$.

Also, check whether the Eigenvectors are orthogonal and orthonormal or not. **[3+3=6]**

7. a) Determine the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ which maps the vectors (1,0,0), (0,1,0), (0,0,1) of R^3 to the vectors (0,0,1), (1,0,0), (0,1,0) respectively. Hence find rank and nullity of T. Also verify Rank-Nullity theorem. **[2+3+2=7]**

b) Find the weight of the minimal spanning tree from the following graph G by Prim's algorithm. **[3]**